DPP No. # A31 (JEE–ADVANCED)

Special DPP's on "Method of Differentiation"

Total Marks : 61		Max. Time : 60 min.
Single choice Objective (no negative marking) Q. 1 to 13	(3 marks, 3 min.)	[39, 39]
Multiple choice objective (no negative marking) Q. 14	(4 marks, 3 min.)	[04, 03]
Subjective Questions (no negative marking) Q.15 to 20	(3 marks, 3 min.)	[18, 18]

Ques. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Mark Obtained																					

1. If $f'(x) = \sqrt{2x^2-1}$ and $y = f(x^2)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

(A) 2 (B) 1 (C) – 2 (D) –1

2. If $f(x) = \log_x (\ell n\, x)$, then $f'(x)$ at $x = e$ is equal to

(A) 1/e (B) e (C) 1 (D) zero

3. Let $y = x^3 - 8x + 7$ and $x = f(t)$. If $\frac{dy}{dt} = 2$ and $x = 3$ at $t = 0$, then $\frac{dx}{dt}$ at $t = 0$ is given by

(A) 1 (B) $\frac{19}{2}$ (C) $\frac{2}{19}$ (D) none of these

4. If $x = \frac{1+t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$, then $x\left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx}$ is equal to

(A) 0 (B) – 1 (C) 1 (D) 2

5. If $x = at^2$, $y = 2at$, then $\frac{d^2y}{dx^2}$ is equal to

(A) $-\frac{1}{t^2}$ (B) $\frac{1}{2at^2}$ (C) $-\frac{1}{t^3}$ (D) $-\frac{1}{2at^3}$

6. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{(1+x)^2}$ (B) $-\frac{1}{(1+x)^2}$ (C) $\frac{1}{\left(1+x^2\right)}$ (D) $\frac{1}{(1+x)}$

7. Let g is the inverse function of f and $f'(x) = \frac{x^{10}}{\left(1+x^2\right)}$. If $g(2) = a$, then $g'(2)$ is equal to

(A) $\frac{a}{2^{10}}$ (B) $\frac{1+a^2}{a^{10}}$ (C) $\frac{a^{10}}{1+a^2}$ (D) $\frac{1+a^{10}}{a^2}$

8. If $ax^2 + 2hxy + by^2 = 0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{y}{x}$ (B) $\frac{x}{y}$ (D) $-\frac{x}{y}$ (D) None of these

9. If $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right)$ = ax + b, then the value of 'a' and 'b' are respectively

(A) 2 and 1 (B) – 2 and 1 (C) 2 and – 1 (D) None of these

10. Let $e^{f(x)} = \ell n\, x$. If g(x) is the inverse function of f(x), then g′ (x) equals to:

(A) e^x (B) $e^x + x$ (C) e^{x+e^x} (D) $e^{x + \ln x}$

11. If $x^m y^n = (x + y)^{m + n}$, then $\frac{dy}{dx}$ is -

(A) $\frac{x+y}{xy}$ (B) xy (C) $\frac{x}{y}$ (D) $\frac{y}{x}$

12. If 8 f(x) + 6 f$\left(\frac{1}{x}\right)$ = x + 5 and y = x^2 f(x), then $\frac{dy}{dx}$ at x = –1 is equal to

(A) 0 (B) $\frac{1}{14}$ (C) $-\frac{1}{14}$ (D) None of these

13. If y = $\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at x = 1 is equal to :

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) 1 (D) $\sqrt{2}$

14. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is equal to

(A) $-\frac{2^y}{2^x}$ (B) $\frac{1}{1-2^x}$ (C) $1 - 2^y$ (D) $\frac{2^x\left(1-2^y\right)}{2^y\left(2^x-1\right)}$

15. Differentiate the following functions with respect to x .

(i) $x^{2/3} + 7e - \frac{5}{x} + 7\tan x$ (ii) $x^2.\ \ell n\, x.\ e^x$ (iii) $\ell n \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

(iv) $\frac{\sin x - x\cos x}{x\sin x + \cos x}$ (v) $\tan\left(\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, $0 < x < \pi$

16. Differentiate the given functions w.r.t.x

(i) $(\ell n\, x)^{\cos x}$ (ii) $x^x - 2^{\sin x}$ (iii) $y = (x\ \ell n\, x)^{\ell n\ \ell n\, x}$

17. If f(x) = 2 ℓn (x – 2) – x^2 + 4x + 1, then find the solution set of the inequality f ′ (x) ≥ 0.

18. Find $\frac{dy}{dx}$ if :

(i) $x = a\left(\cos t + \frac{1}{2}\ell n \tan^2\frac{t}{2}\right)$ and y = a sin t.

(ii) $x = \sin t\sqrt{\cos 2t}$ and $y = \cos t\sqrt{\cos 2t}$

19. Find $\frac{dy}{dx}$, when x and y are connected by the following relations

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ (ii) $xy + xe^{-y} + y \cdot e^x = x^2$

20. Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = \tan^{-1}\frac{4x}{1+5x^2} + \tan^{-1}\frac{2+3x}{3-2x}$, $(0 < x < 1)$ (ii) $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $(0 < x < 1)$

(iii) $y = \sin^{-1}\sqrt{\frac{1+x}{2}}$, $(-1 < x < 1)$

DPP No. # A32 (JEE–ADVANCED)

Special DPP's on "Method of Differentiation"

Total Marks : 64		**Max. Time : 60 min.**
Single choice Objective (no negative marking) Q.1 to 8	**(3 marks, 3 min.)**	**[24, 24]**
Multiple choice objective (no negative marking) Q. 9 to 12	**(4 marks, 3 min.)**	**[16, 12]**
Subjective Questions (no negative marking) Q.13 to 20	**(3 marks, 3 min.)**	**[24, 24]**

Ques. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Mark obtained																					

1. Let f(x) be a polynomial in x. Then the second derivative of $f(e^x)$ w.r.t. x is

(A) $f''(e^x).\ e^x + f'(e^x)$ (B) $f''(e^x).\ e^{2x} + f'(e^x).\ e^{2x}$

(C) $f''(e^x)\ e^{2x}$ (D) $f''(e^x).\ e^{2x} + f'(e^x).\ e^x$

2. If f(x), g(x), h(x) are polynomials in x of degree 2 and $F(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}$, then $F'(x)$ is equal to

(A) 1 (B) 0 (C) –1 (D) f(x) . g(x) . h(x)

3. **Statement - 1** Let $f : [0, \infty) \to [0, \infty)$ be a function defined by $y = f(x) = x^2$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = 1$.

Statement - 2 $\frac{d^2y}{dx^2} = -\frac{d^2x}{dy^2} \cdot \left(\frac{dy}{dx}\right)^3$

(A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1

(C) STATEMENT-1 is true, STATEMENT-2 is false

(D) STATEMENT-1 is false, STATEMENT-2 is true

(E) Both STATEMENTS are false

4. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

(A) 1 (B) log 2 (C) – log 2 (D) – 1

5. Let $f : (-1, 1) \to \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0)$.

(A) – 4 (B) 0 (C) – 2 (D) 4

6. $\dfrac{d^2x}{dy^2} =$

(A) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ (C) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$ (D) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$

7. If $y = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$, then $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx}$ is equal to

(A) 4 (B) 3 (C) 1 (D) 0

8. If $y^2 = P(x)$ is a polynomial of degree 3, then $2\left(\dfrac{d}{dx}\right)\left(y^3.\dfrac{d^2y}{dx^2}\right)$ is equal to

(A) $P'''(x) + P'(x)$ (B) $P''(x).\, P'''(x)$ (C) $P(x).\, P'''(x)$ (D) a constant

9. The differential coefficient of $\sin^{-1}\dfrac{t}{\sqrt{1+t^2}}$ w.r.t. $\cos^{-1}\dfrac{1}{\sqrt{1+t^2}}$ is

(A) $1 \ \forall\, t > 0$ (B) $-1 \ \forall\, t < 0$ (C) $1 \ \forall\, t \in R$ (D) none of these

10. If $f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\dfrac{d}{dx}\{f_n(x)\}$ is equal to

(A) $f_n(x).\dfrac{d}{dx}\{f_{n-1}(x)\}$ (B) $f_n(x).\, f_{n-1}(x)$

(C) $f_n(x).\, f_{n-1}(x) \ldots\ldots.\, f_2(x).\, f_1(x)$ (D) none of these

11. If $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$, then

(A) $f(-2) = 0$ (B) $f'(-1/2) = 0$ (C) $f'(-1) = -2$ (D) $f''(0) = 4$

12. If f is twice differentiable such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x)$ is a twice differentiable function such that $h'(x) = (f(x))^2 + (g(x))^2$. If $h(0) = 2$, $h(1) = 4$, then the equation $y = h(x)$ represents

(A) a curve of degree 2 (B) a curve passing through the origin

(C) a straight line with slope 2 (D) a straight line with y intercept equal to 2.

13. If $y = \cfrac{x}{a+\cfrac{x}{b+\cfrac{x}{a+\ldots\ldots\ldots\ldots\infty}}}$, then find $\dfrac{dy}{dx}$

14. It is known for $x \neq 1$ that $1 + x + x^2 + \ldots.+x^{n-1} = \dfrac{1-x^n}{1-x}$, hence find the sum of the series $S = 1 + 2x + 3x^2 \ldots.+ (n + 1)\, x^n$.

15. If $y = a^{x^{a^{x^{\cdot^{\cdot^{\cdot^{\infty}}}}}}}$, then prove that $\dfrac{dy}{dx} = \dfrac{y^2 \log_e y}{x(1-y\log_e x \quad \log_e y)}$

16. Differentiate

(i) $\tan^{-1}\left(\frac{1+2x}{1-2x}\right)$ w.r.t. $\sqrt{1+4x^2}$ (ii) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}(x)$

17. (i) If $e^y (x + 1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

(ii) If $y = \sin(2\sin^{-1} x)$, show that $(1 - x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx} - 4y$.

18. If $y = A e^{-kt} \cos(pt + c)$, then prove that $\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2 y = 0$, where $n^2 = p^2 + k^2$.

19. If $e^{x+y} = xy$, then show that $\frac{d^2y}{dx^2} = \frac{-y\left((x-1)^2+(y-1)^2\right)}{x^2(y-1)^3}$.

20. If $y = x\,\ell n((ax)^{-1} + a^{-1})$, prove that $x(x+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = y - 1$.

DPP No. # A33 (JEE–ADVANCED)

Special DPP's on "Method of Differentiation"

Total Marks : 63		**Max. Time : 60 min.**	
Single choice Objective (no negative marking) Q.1 to 10	**(3 marks, 3 min.)**	**[30, 30]**	
Multiple choice objective (no negative marking) Q.11 to 13	**(4 marks, 3 min.)**	**[12, 09]**	
Subjective Questions (no negative marking) Q. 14 to 20	**(3 marks, 3 min.)**	**[21, 21]**	

Ques. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
Mark obtained																					

1. If $y = \sin^{-1}\left(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right)$ and $\frac{dy}{dx} = \frac{1}{2\sqrt{x(1-x)}} + p$, then p is equal to

(A) 0 (B) $\frac{1}{\sqrt{1-x}}$ (C) $\sin^{-1}\sqrt{x}$ (D) $\frac{1}{\sqrt{1-x^2}}$

2. If $u = ax + b$, then $\frac{d^n}{dx^n}(f(ax+b))$ is equal to

(A) $\frac{d^n}{du^n}(f(u))$ (B) $a\frac{d^n}{du^n}(f(u))$ (C) $a^n\frac{d^n}{du^n}(f(u))$ (D) $a^{-n}\frac{d^n}{dx^n}(f(u))$

3. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is equal to

(A) e^x (B) $-\frac{e^x}{(1+e^x)^3}$ (C) $-\frac{e^x}{(1+e^x)^2}$ (D) $\frac{1}{(1+e^x)^2}$

4. If y is a function of x and $\ell n(x + y) - 2xy = 0$, then the value of $y'(0)$ is equal to

(A) 1 (B) –1 (C) 2 (D) 0

5. Let S denote the set of all polynomials P(x) of degree ≤ 2 such that P(1) = 1, P(0) = 0 and $P'(x) > 0 \ \forall \ x \in [0, 1]$, then

(A) $S = \phi$
(B) $S = \{(1 - a)x^2 + ax \ ; \ 0 < a < 2\}$
(C) $S = \{(1 - a)\, x^2 + ax \ ; \ 0 < a < 1\}$
(D) $S = \{(1 - a)\, x^2 + ax \ ; \ 0 < a < \infty\}$

6. If x cos y + y cos x = π then the value of $y''(0)$ is equal to

(A) π
(B) $-\pi$
(C) 1
(D) 0

7. If $f''(x) = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that F(5) = 5, then F(10) is equal to

(A) 5
(B) 10
(C) 0
(D) 15

8. Let g(x) = log f(x), where f(x) is a twice differentiable positive function on $(0, \infty)$ such that $f(x + 1) = x\, f(x)$. Then, $g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right)$ is equal to, for N = 1, 2, 3, ...

(A) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N-1)^2}\right\}$
(B) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N-1)^2}\right\}$
(C) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N+1)^2}\right\}$
(D) $4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N+1)^2}\right\}$

9. The area of the closed figure bounded by y = x, y = – x and the tangent to the curve $y = \sqrt{x^2 - 5}$ at the point (3, 2) is

(A) 5
(B) $\frac{15}{2}$
(C) 10
(D) $\frac{35}{2}$

10. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1 - 9x^3}\right)$ is $\sqrt{x}$. g(x), then g(x) equals :

(A) $\frac{9}{1 + 9x^3}$
(B) $\frac{3x\sqrt{x}}{1 - 9x^3}$
(C) $\frac{3x}{1 - 9x^3}$
(D) $\frac{3}{1 + 9x^3}$

11. If $\sqrt{x^2 + y^2} = e^t$ where $t = \sin^{-1}\left(\frac{y}{\sqrt{x^2 + y^2}}\right)$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{x - y}{x + y}$
(B) $\frac{x + y}{x - y}$
(C) $\frac{y - x}{y + x}$
(D) $\frac{x - y}{2x + y}$

12. Let $f(x) = x \sin \pi x$, x > 0. Then for all natural numbers n, $f'(x)$ vanishes at

(A) a unique point in the interval $\left(n, \ n + \frac{1}{2}\right)$
(B) a unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$
(C) a unique point in the interval (n, n +1)
(D) two points in the interval (n, n +1)

13. The functions $u = e^x \sin x$; $v = e^x \cos x$ satisfy the equation

(A) $v\dfrac{du}{dx} - u\dfrac{dv}{dx} = u^2 + v^2$ (B) $\dfrac{d^2u}{dx^2} = 2v$

(C) $\dfrac{d^2v}{dx^2} = -2u$ (D) $\dfrac{du}{dx} + \dfrac{dv}{dx} = 2v$

14. If P_n is the sum of a GP upto n terms. Show that $(1 - r)\dfrac{dP_n}{dr} = n \,.\, P_{n-1} - (n-1)P_n$.

15. If $\cos\dfrac{x}{2}\,.\,\cos\dfrac{x}{2^2}\,.\,\cos\dfrac{x}{2^3}\ldots\infty = \dfrac{\sin x}{x}$, then find the value of $\dfrac{1}{2^2}\sec^2\dfrac{x}{2} + \dfrac{1}{2^4}\sec^2\dfrac{x}{2^2} + \dfrac{1}{2^6}\sec^2\dfrac{x}{2^3} + \ldots\ldots\infty$

16. Show that the substitution $z = \ell n\left(\tan\dfrac{x}{2}\right)$ changes the equation $\dfrac{d^2y}{dx^2} + \cot x\dfrac{dy}{dx} + 4y\,\mathrm{cosec}^2x = 0$ to $(d^2y/dz^2) + 4y = 0$.

17. If $F(x) = f(x).\, g(x)$ and $f'(x).\, g'(x) = c$, prove that $\dfrac{F''}{F} = \dfrac{f''}{f} + \dfrac{g''}{g} + \dfrac{2c}{fg}$ and $\dfrac{F'''}{F} = \dfrac{f'''}{f} + \dfrac{g'''}{g}$

18. If $y = \dfrac{ax^2}{(x-a)(x-b)(x-c)} + \dfrac{bx}{(x-b)(x-c)} + \dfrac{c}{x-c} + 1$, then prove that $\dfrac{y'}{y} = \dfrac{1}{x}\left(\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}\right)$

19. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then show that $(x^2+4)\left(\dfrac{dy}{dx}\right)^2 = n^2(y^2+4)$

20. If $(a + bx)\, e^{\frac{y}{x}} = x$, then prove that $x^3\dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$

ANSWER KEY

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15. (i) $\frac{2}{3x^{\frac{1}{3}}} + \frac{5}{x^2} + 7\sec^2 x$

(ii) $e^x\, x\, (2\,\ell n\, x + 1 + x\,\ell n\, x)$

(iii) $\sec x$

(iv) $\frac{x^2}{(x \sin x + \cos x)^2}$

(v) $\frac{1}{2}\sec^2 \frac{x}{2}$

16. (i) $(\ell n\, x)^{\cos x}\left(\frac{\cos x}{x \ell n x} - \sin x \ell n(\ell n x)\right)$

(ii) $x^x (1 + \ell n\, x) - \ell n 2 \,.\, 2^{\sin x} \,.\, \cos x$

(iii) $(x\,\ell n\, x)^{\ell n\, \ell n\, x}$.

$$\frac{1}{x}\left(1 + \ell n\,(\ell n x)\left(1 + \frac{2}{\ell n\, x}\right)\right)$$

17. (2, 3]

18. (i) tan t (ii) – tan 3t

19. (i) $-\frac{ax+hy+g}{hx+by+f}$

(ii) $\frac{2x - y - e^{-y} - e^x y}{x - xe^{-y} + e^x}$

20. (i) $\frac{dy}{dx} = \frac{5}{(1+25x^2)}$

(ii) $\frac{dy}{dx} = -\frac{2}{1+x^2}$

(iii) $\frac{1}{2\sqrt{1-x^2}}$

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13. $\frac{b}{ab + 2ay}$

14. $\frac{(n+1)x^{n+2} - (n+2)x^{n+1} + 1}{(1-x)^2}$

16. (i) $\frac{1}{2x\sqrt{1+4x^2}}$ (ii) $\frac{1}{2}$

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15. $\text{cosec}^2\, x - (1/x^2)$